

AP Calculus BC Summer Assignment

Complete all problems and show work on separate paper.

Section I

Find the derivative:

1) $y = (x^4 - 3x^2 + 5)^3$

2) $y = \cos(\tan x)$

3) $y = 2x\sqrt{x^2 + 1}$

4) $y = \left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$

5) If $f(t) = \sqrt{4t+1}$ find $f''(2)$

6) If $g(\theta) = \theta \sin \theta$, find $g''(\pi/6)$

7) Find the equations of the tangent and normal lines to the curve at the point: (2, 1)

$$x^2 + 4xy + y^2 = 13$$

8) Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope of 1.

9) A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3, t \geq 0$.

- Find the velocity and acceleration functions.
- When is the particle moving upward and when is it moving downward?
- Find the distance that the particle travels in the time interval $[0, 3]$.
- Graph all three functions over the same time interval.
- When is the particle speeding up? Slowing down?

12) A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

13) A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cubic cm/s, how fast is the water level rising when the water is 5 cm deep?

Section II

14) Sketch a function that satisfies the condition: $f(0) = 0$, f is continuous and even,

$$f'(x) = 2x \text{ if } 0 < x < 1, \quad f'(x) = -1 \text{ if } 1 < x < 3, \quad f'(x) = 1 \text{ if } x > 3$$

15) Sketch the curve using calculus (first, second derivative): $y = \frac{1}{x(x-3)^2}$

16) Find $f(x)$ if: $f'(x) = 8x - 3\sec^2 x$

17) Find $f(x)$ if: $f'(x) = 2x - 3\sin t$

18) Find the point on the hyperbola $xy = 8$ that is closest to the point (3, 0).

19) Differentiate: $F(x) = \int_{2x}^{3x+1} \sin(t^4) dt$

20) Differentiate: $y = (\cos x)^x$

Section III

Integrate:

21) $\int \sin \pi t \cos \pi t dt$

22) $\int_0^2 y^2 \sqrt{1+y^3} dy$

23) $\int \frac{1}{16+t^2} dt$

24) $\int \frac{\cos(\ln x)}{x} dx$

25) $\int \frac{x+1}{x^2+2x} dx$

26) $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

27) $\int x^2 \sin x dx$

28) $\int \ln(2x+1) dx$

29) $\int t \sec^2 2t dt$

30) $\int \frac{(\ln x)^2}{x^3} dx$

31) $\int \arctan 4x dx$

32) $\int e^x \sin x dx$

Find the area enclosed by the curves. Draw a picture! No calculators!

33) $y = \sin x, y = 0, x = \pi/2, x = \pi$

34) $y = x^2 - 2x, y = x + 4$

35) $y = x^3 - x, y = 3x$

Section IV

Find the volume of the solid rotated about the given line, using the most convenient method:

Sketch all regions!! NO Calculators!

37) $y = 1 + \sec x, y = 3$, about $y = 1$

38) $y = 4(x - 2)^2, y = x^2 - 4x + 7$, about the x -axis

39) $x = y^2, x = 1$, about $x = 1$

40) $y^2 = x, x = 2y$, about the y -axis

41) $x = 1 + (y - 2)^2, x = 2$, about the x - axis

Set up, but do not evaluate, an integral for the solid:

42) $y = \tan^3 x, y = 1, x = 0$ about $y = 1$

43) $x^2 - y^2 = 1, x = 3$, about $x = -2$