

# AP Calculus BC Summer Assignment

Complete all problems and show work on separate paper.

## Section I

Find the derivative:

1)  $y = (x^4 - 3x^2 + 5)^3$

2)  $y = \cos(\tan x)$

3)  $y = 2x\sqrt{x^2 + 1}$

4)  $y = \left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$

5) If  $f(t) = \sqrt{4t+1}$  find  $f''(2)$

6) If  $g(\theta) = \theta \sin \theta$ , find  $g''(\pi/6)$

7) Find the equations of the tangent and normal lines to the curve at the point: (2, 1)

$$x^2 + 4xy + y^2 = 13$$

8) Find the points on the ellipse  $x^2 + 2y^2 = 1$  where the tangent line has slope of 1.

9) A particle moves on a vertical line so that its coordinate at time  $t$  is  $y = t^3 - 12t + 3, t \geq 0$ .

a) Find the velocity and acceleration functions.

b) When is the particle moving upward and when is it moving downward?

c) Find the distance that the particle travels in the time interval  $[0, 3]$ .

d) Graph all three functions over the same time interval.

e) When is the particle speeding up? Slowing down?

12) A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

13) A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cubic cm/s, how fast is the water level rising when the water is 5 cm deep?

### Section II

14) Sketch a function that satisfies the condition:  $f(0) = 0$ ,  $f$  is continuous and even,

$$f'(x) = 2x \text{ if } 0 < x < 1, \quad f'(x) = -1 \text{ if } 1 < x < 3, \quad f'(x) = 1 \text{ if } x > 3$$

15) Sketch the curve using calculus (first, second derivative):  $y = \frac{1}{x(x-3)^2}$

16) Find  $f(x)$  if:  $f'(x) = 8x - 3\sec^2 x$

17) Find  $f(x)$  if:  $f'(x) = 2x - 3\sin t$

18) Find the point on the hyperbola  $xy = 8$  that is closest to the point (3, 0).

19) Differentiate:  $F(x) = \int_{2x}^{3x+1} \sin(t^4) dt$

20) Differentiate:  $y = (\cos x)^x$

### Section III

Integrate:

21)  $\int \sin \pi t \cos \pi t dt$

22)  $\int_0^2 y^2 \sqrt{1+y^3} dy$

23)  $\int \frac{1}{16+t^2} dt$

24)  $\int \frac{\cos(\ln x)}{x} dx$

25)  $\int \frac{x+1}{x^2+2x} dx$

26)  $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

27)  $\int x^2 \sin x dx$

28)  $\int \ln(2x+1) dx$

29)  $\int t \sec^2 2t dt$

30)  $\int \frac{(\ln x)^2}{x^3} dx$

31)  $\int \arctan 4x dx$

32)  $\int e^x \sin x dx$

Find the area enclosed by the curves. Draw a picture! No calculators!

33)  $y = \sin x, y = 0, x = \pi/2, x = \pi$

34)  $y = x^2 - 2x, y = x + 4$

35)  $y = x^3 - x, y = 3x$

#### Section IV

Find the volume of the solid rotated about the given line, using the most convenient method:

Sketch all regions!! NO Calculators!

37)  $y = 1 + \sec x, y = 3$ , about  $y = 1$

38)  $y = 4(x - 2)^2, y = x^2 - 4x + 7$ , about the  $y$ -axis

39)  $x = y^2, x = 1$ , about  $x = 1$

40)  $y^2 = x, x = 2y$ , about the  $y$ -axis

41)  $x = 1 + (y - 2)^2, x = 2$ , about the  $x$  - axis

Set up, but do not evaluate, an integral for the solid:

42)  $y = \tan^3 x, y = 1, x = 0$  about  $y = 1$

43)  $x^2 - y^2 = 1, x = 3$ , about  $x = -2$